# Cramér–von Mises distance: Probabilistic interpretation, confidence intervals, and neighborhood-of-model validation

## Norbert Henze

*Karlsruhe Institute of Technology, Karlsruhe, Germany*

(joint work with Ludwig Baringhaus, University of Hannover, Germany)

**Abstract:** We give a probabilistic interpretation of the Cramér–von Mises distance

$$\Delta(F, F_0) = \int (F - F_0)^2 \mathrm{d}F_0$$

between continuous distribution functions $F$ and $F_0$. If $F$ is unknown, we construct an asymptotic confidence interval for $\Delta(F, F_0)$ based on a random sample from $F$. Moreover, for given $F_0$ and some value $\Delta_0 > 0$, we propose an asymptotic equivalence test of the hypothesis that $\Delta(F, F_0) \geq \Delta_0$ against the alternative $\Delta(F, F_0) < \Delta_0$. If such a 'neighborhood-of-$F_0$ validation test', carried out at a small asymptotic level, rejects the hypothesis, there is evidence that $F$ is within a distance $\Delta_0$ of $F_0$. As a neighborhood-of-exponentiality test shows, the method may be extended to the case that $H_0$ is composite.